Data Assimilation in a Sea Ice Model

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Motivation

1. Availability of sea ice thickness data.
2. Lack of understanding of sea ice rheology
3. Necessity to proceed from descriptive to predictive sea ice models.

Model Design Requirements

1. Variational formulation for complex rheologies.
2. Accurate mesh refinement and adaptive meshes.
3. Easy coupling with FEOM.

Purpose of assimilation

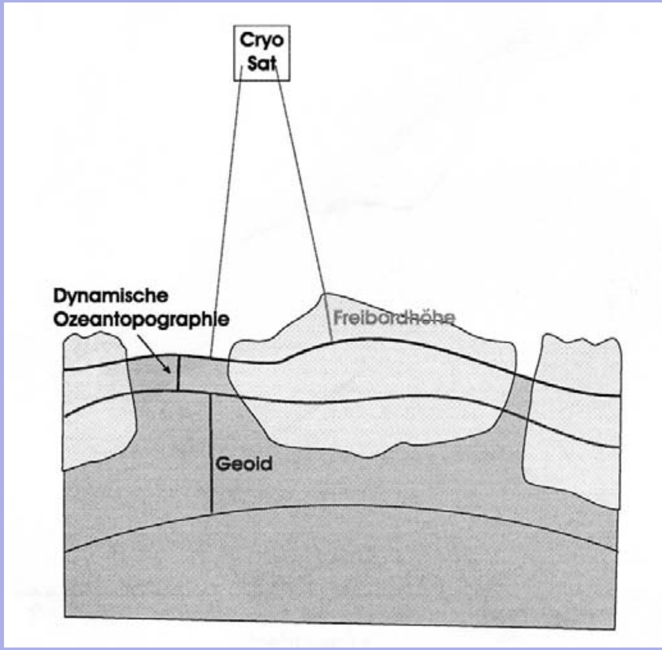
Interpolation/extrapolation into regions/ reanalysis variables unobserved (state estimation,)

Initialisation for prediction

Systematic improvement of model performance

Quality check for measurements

Measurement of sea ice thickness



New AWI sea ice model

- AWI thermodynamic Sea Ice Model

Recoded in Fortran 90 in a manner to suite FE formalism.

- Transport of ice mass and compactness

Implicit backward Euler stabilized FE scheme.

- Ice dynamics with non-linear rheology

1. Viscous-Plastic (Hibler 1979).
2. Elastic-Viscous-Plastic (Hunke and Dukowicz, 1997).

Ice Dynamics

Governing equations of dynamics

$$m\frac{\partial \vec{u}_i}{\partial t} + mf\vec{k}\times\vec{u}_i = -mg\vec{\nabla}\zeta + \vec{\tau}_a + \vec{\tau}_w + \mathbf{F}$$

F - rheology

m – ice mass; *m*=*ah*, where *h* is the ice thickness and *a* is the ice compactness

$$\vec{\tau}_w = \rho_w C_w \left|\Delta\vec{u}_{iw}\right|(\Delta\vec{u}_{iw}\cos\varphi + \vec{k}\times\Delta\vec{u}_{iw}\sin\varphi)$$

Transport of mass and compactness

$$\partial_t m + div(\vec{u}m) = 0, \quad \partial_t a + div(\vec{u}a) = 0$$

Rheology

$$\mathbf{F} = \nabla \cdot \sigma$$

$$\begin{pmatrix} F_\lambda \\ F_\theta \end{pmatrix} = \frac{1}{R\sin\theta}\begin{pmatrix} \frac{\partial}{\partial\lambda}\sigma_{11} + \frac{\partial}{\partial\theta}(\sigma_{12}\sin\theta) + \sigma_{12}\cos\theta \\ \frac{\partial}{\partial\lambda}\sigma_{12} + \frac{\partial}{\partial\theta}(\sigma_{22}\sin\theta) - \sigma_{11}\cos\theta \end{pmatrix}$$

Deformation Rates Tensor

$$\dot{\varepsilon}_{11} = \frac{1}{R\sin\theta}\left(\frac{\partial u}{\partial\lambda} + v\cos\theta\right)$$

$$\dot{\varepsilon}_{22} = \frac{1}{R}\frac{\partial v}{\partial\theta}$$

$$\dot{\varepsilon}_{12} = \frac{1}{2R}\left(\sin\theta\frac{\partial}{\partial\theta}\left(\frac{u}{\sin\theta}\right) + \frac{1}{\sin\theta}\frac{\partial v}{\partial\lambda}\right)$$

viscous-plastic rheology

$$\sigma = \zeta D_I \mathbf{I} + 2\eta \mathbf{D}' - \frac{1}{2} P \mathbf{I}$$

D – matrix composed of deformation rates

$$D_I = tr\,\mathbf{D}$$ – divergence

$$\mathbf{D}' = \mathbf{D} - \frac{1}{2} D_I \mathbf{I}$$ – shear

$$D_{II}^2 = tr\,\mathbf{D}'\mathbf{D}'$$

$$\Delta^2 = D_I^2 + \left(D_{II}/e^2\right)^2$$

pressure and moduli

$$P = P^* h e^{-c(1-a)}$$

$$\zeta = \frac{P}{2\Delta + \Delta_{\min}}$$

$$\eta = \zeta / e^2$$

ice pressure regularisation by M. Harder (AWI)

$$\boxed{\widetilde{P} = \frac{P\Delta}{\Delta + \Delta_{\min}}}$$

viscous-plastic rheology

$$\sigma = \zeta D_I \mathbf{I} + 2\eta \mathbf{D}' - \frac{1}{2} P \mathbf{I}$$

$$D_I = tr\,\mathbf{D}$$

$$D_{II}^2 = tr\,\mathbf{D}'\mathbf{D}'$$

$$\mathbf{D}' = \mathbf{D} - \frac{1}{2} D_I \mathbf{I}$$

$$\zeta = \frac{P}{2\Delta}$$

$$\eta = \zeta / e^2$$

$$\Delta^2 = D_I^2 + \left(D_{II} / e^2 \right)^2$$

$$\sigma_{11} = (\zeta - \eta) D_I + \frac{2\eta}{R \sin\theta} \left(\frac{\partial u}{\partial \lambda} + v \cos\theta \right) - \frac{P}{2}$$

$$\sigma_{12} = \frac{\eta}{R} \left(\sin\theta \frac{\partial}{\partial\theta} \left(\frac{u}{\sin\theta} \right) + \frac{1}{\sin\theta} \frac{\partial v}{\partial\lambda} \right)$$

$$\sigma_{22} = (\zeta - \eta) D_I + \frac{2\eta}{R} \frac{\partial v}{\partial\theta} - \frac{P}{2}$$

ice pressure regularisation by M. Harder (AWI)

$$\boxed{\widetilde{P} = \frac{P\Delta}{\Delta + \Delta_{min}}}$$

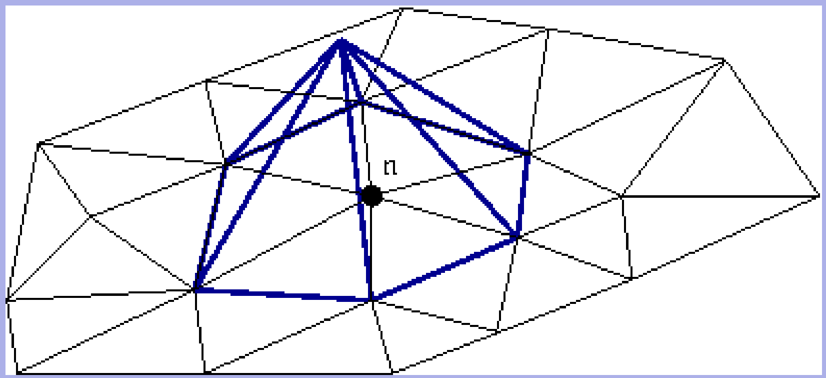
realized as explicit time scheme – severe computational cost.

("internal" time step 1-3 sec)

iterative solution – no accuracy control

In FE: (a) fields are expanded in series

$$\vec{u} = \sum_{1}^{N} \vec{u}_i \varphi_i; \quad m = \sum_{1}^{N} m_i \varphi_i; \quad a = \sum_{1}^{N} a_i \varphi_i;$$



φ_n

(b) Residual are required to be orthogonal to the set of functions.

$$\int \vec{F} \vec{\varphi}_j \, dS = -\int \sigma \nabla \vec{\varphi}_j \, dS$$

(c) Equations on coefficients are solved. Coefficients coincide with nodal values of fields

Viscous-Plastic Rheology

Realized as explicit time scheme – high computational cost.

("internal" time step 1-3 sec)

Iterative solution – no accuracy control.

Elastic-Viscous-Plastic Rheology

- Efficient Numerics – Accuracy Control

("Internal" time step 1-3 min)

- Explicit Time Scheme – Easy Parallelization.

$$\partial_t \sigma_1 + \sigma_1 / 2T + P / 2T = (P / 2T\Delta) D_I$$

$$\partial_t \sigma_2 + \sigma_2 /(2e^2 T) = (P / 2T\Delta) D_T$$

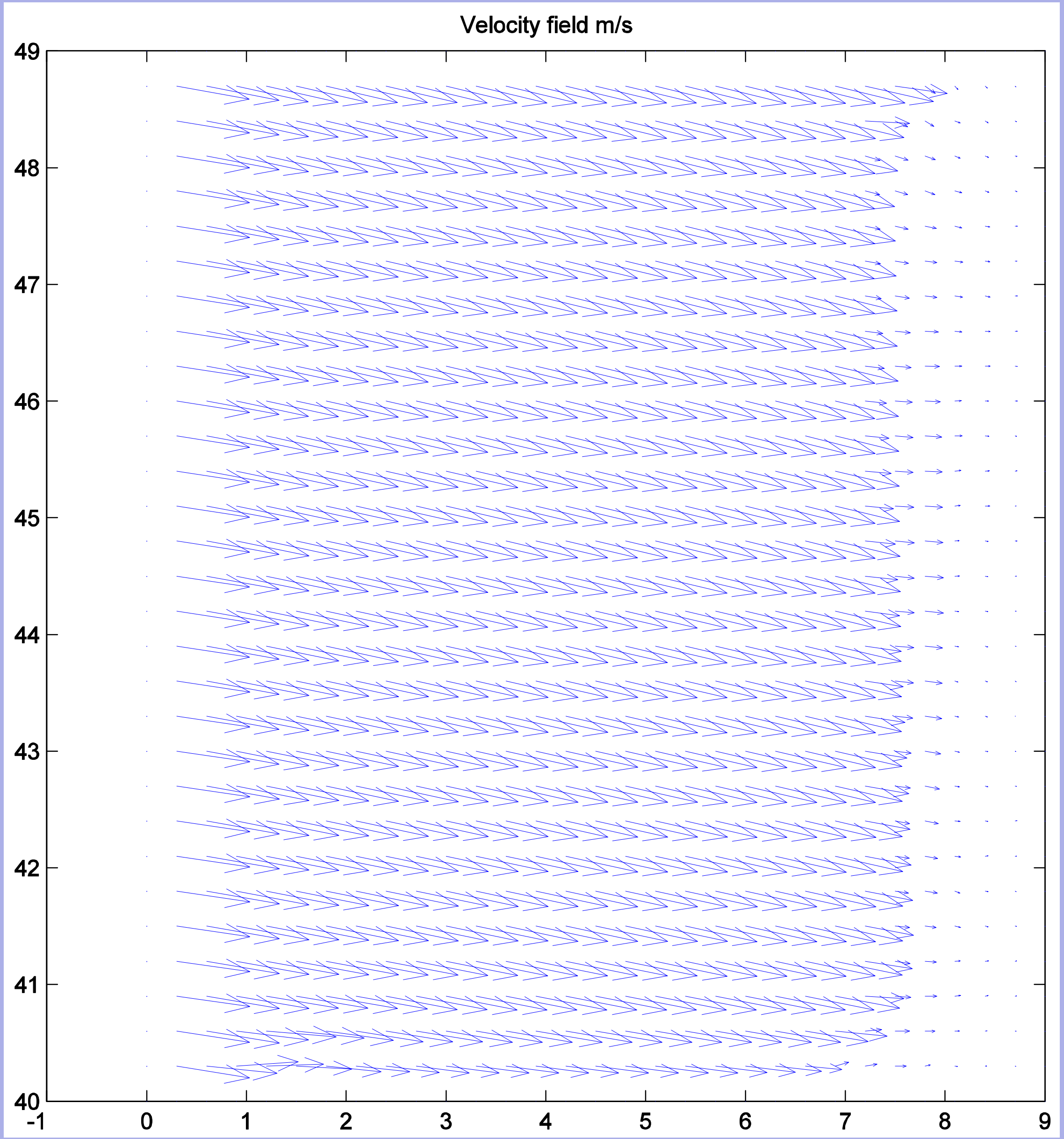
$$\partial_t \sigma_{12} + \sigma_{12} /(2e^2 T) = (P / 2T\Delta) D_S$$

$$\sigma_1 = \sigma_{11} + \sigma_{22}; \sigma_2 = \sigma_{11} - \sigma_{22};$$

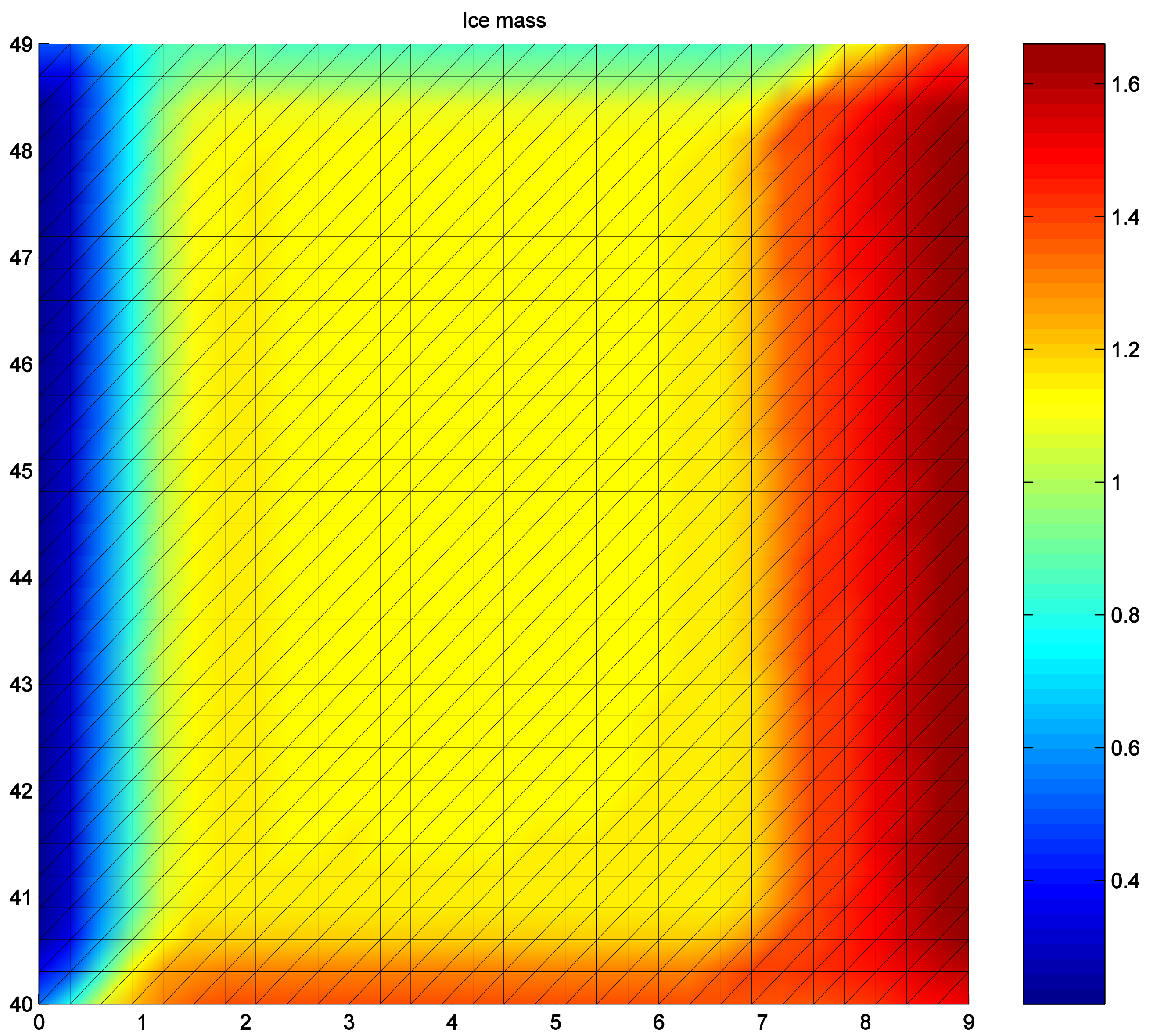
$$D_T = \varepsilon_{11} - \varepsilon_{22}; D_S = \varepsilon_{12}$$

T is the relaxation parameter, T=(1/3)Δt

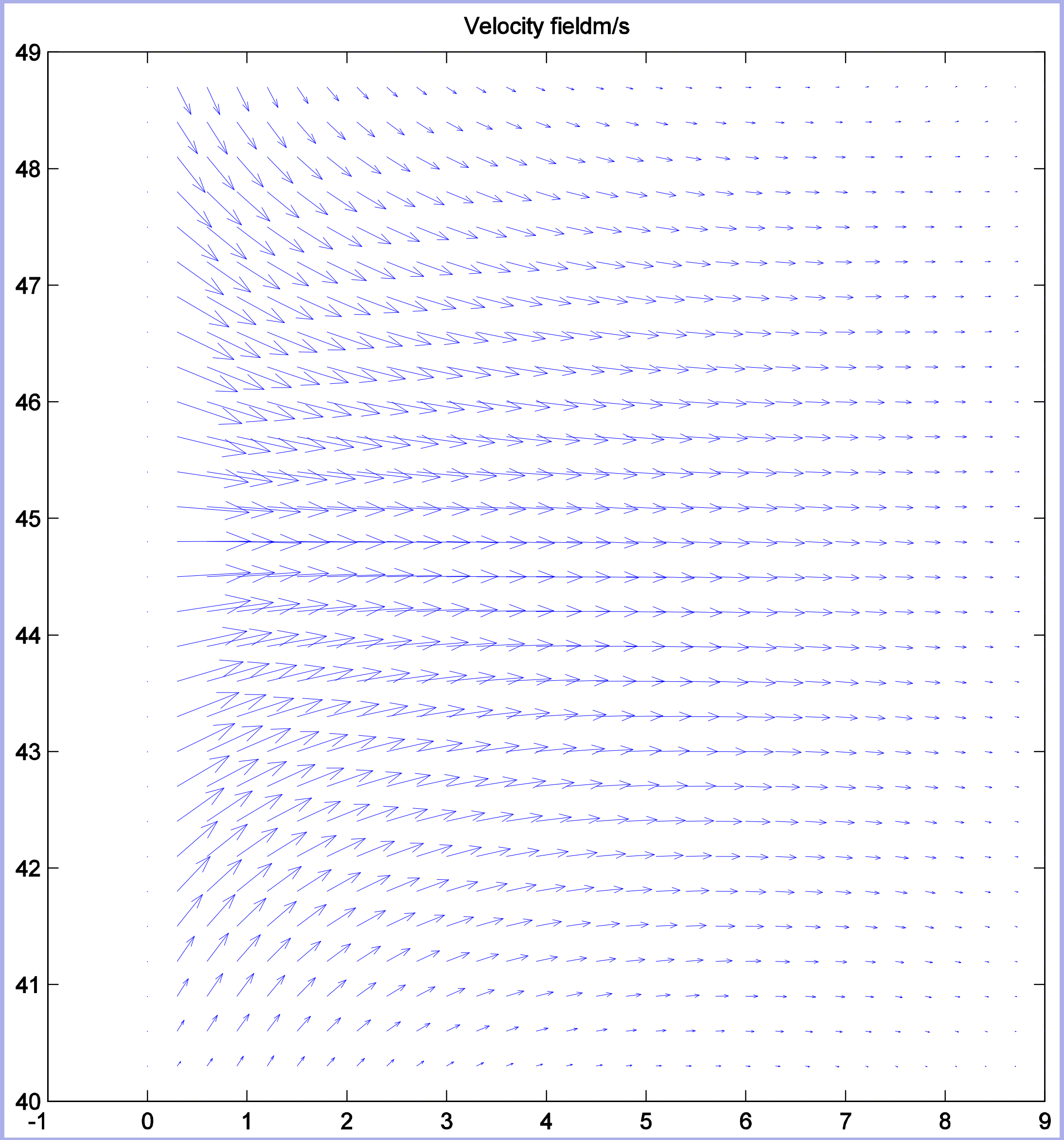
Ice Drift Velocity after 5 days of Integration



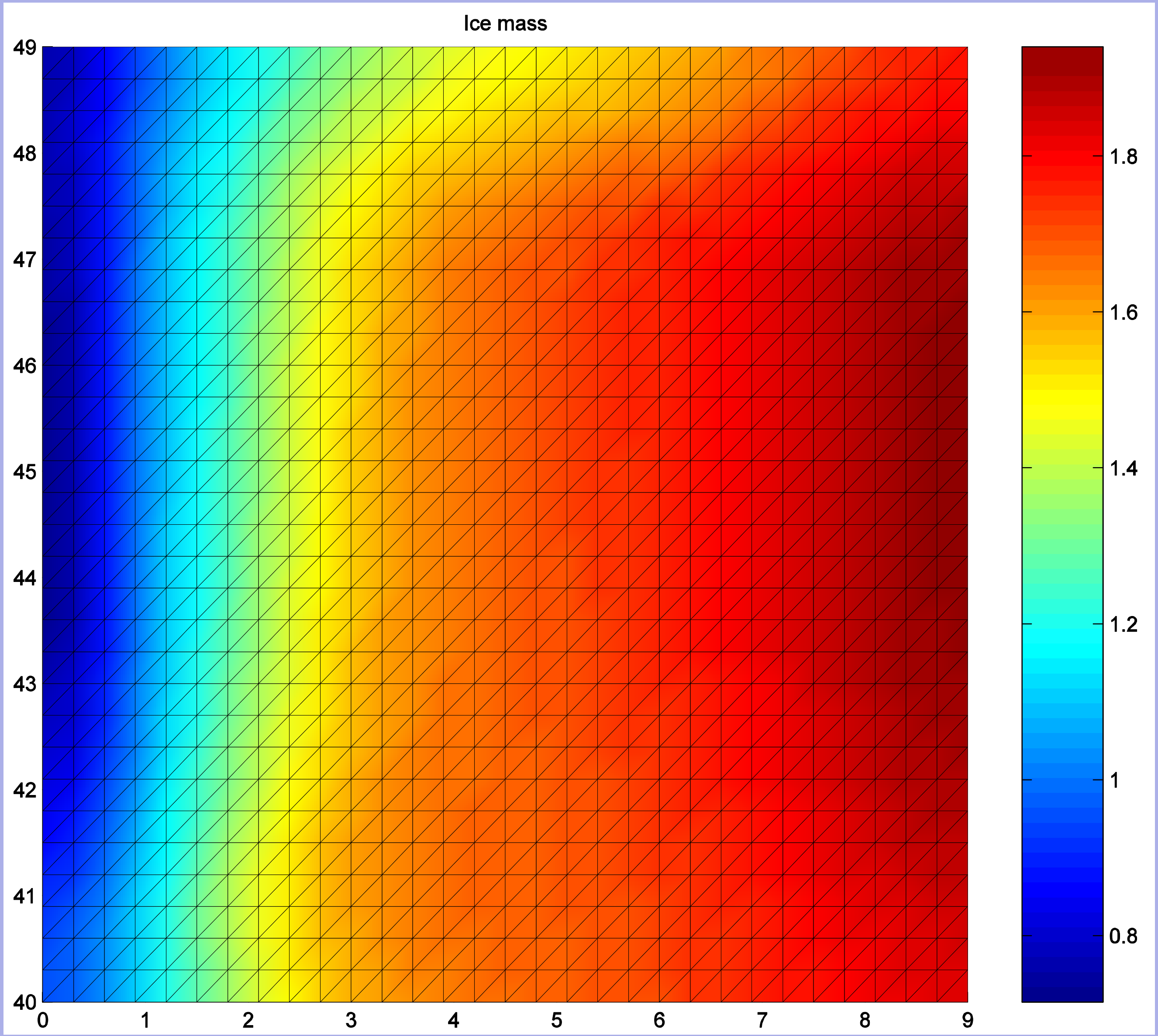
Ice Mass after 5 days of Integration



Ice Drift Velocity after 30 days of Integration



Ice Mass after 30 days of Integration



Work Plan

•Realistic simulations with real Arctic Ocean geometry

•Determination of 'free' parameters (e.g. c, P or P*, etc.) by assimilation of sea ice thickness / distribution data and velocity estimates with the SIR Filter

. Perspectives

•Ridging schemes implementation

•Mesh refinement for complex geometries (e.g. Canadian Archipelago).

•Adaptive meshes for ice condition forecast.

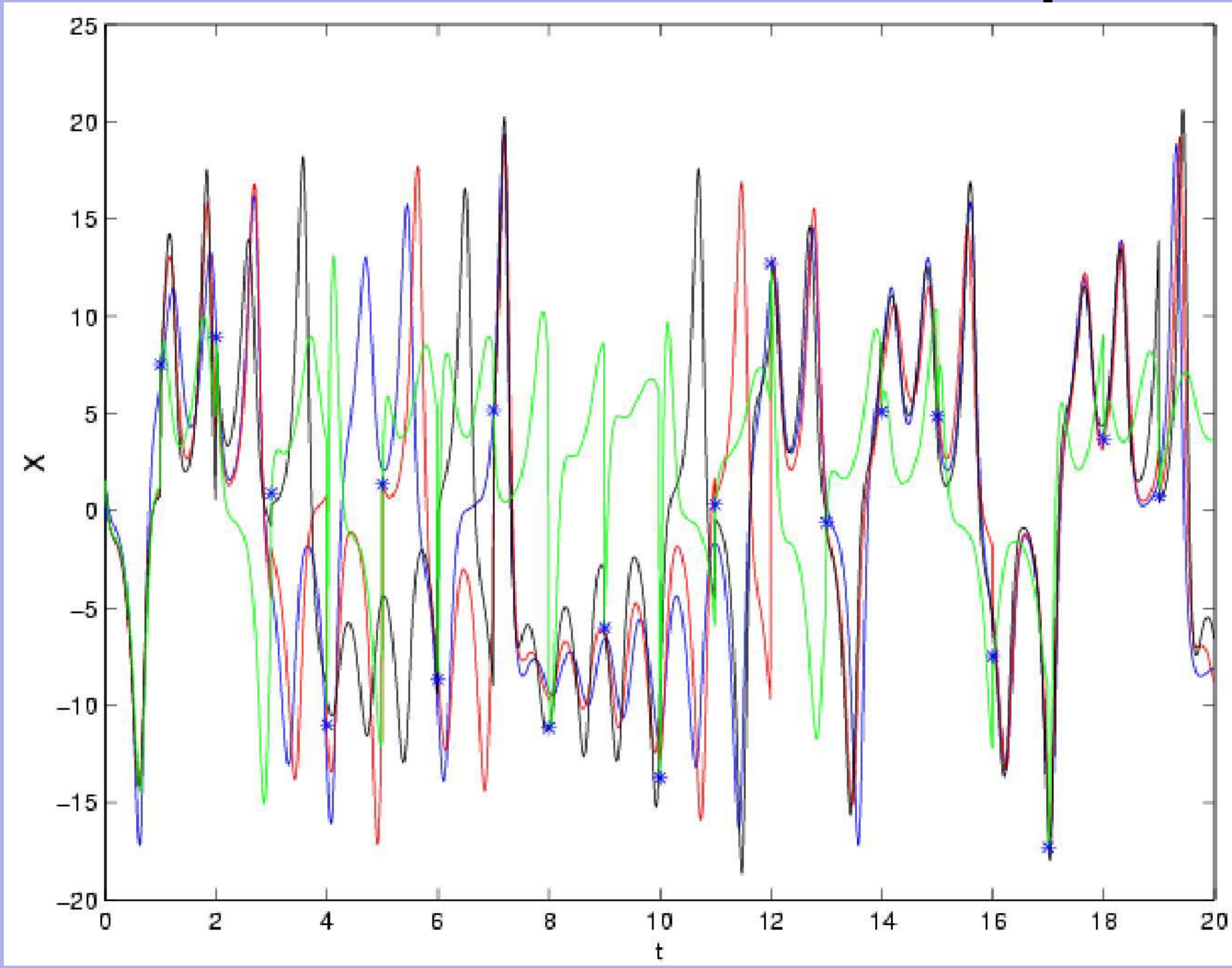
•Pre-operational model set-up with forecast and verification by helicopter measurements of ice thickness.

SIR Filter example

Non linear 'chaotic' Lorenz equations.

- A) determination of Lorenz model state (x,y,z)
- B) determination of Lorenz model parameters (gamma, r)
- Comparison with Ensemble Kalman Filter.
- Performance with small (250) or large (1000) ensemble.

Lorenz model x component (obs)



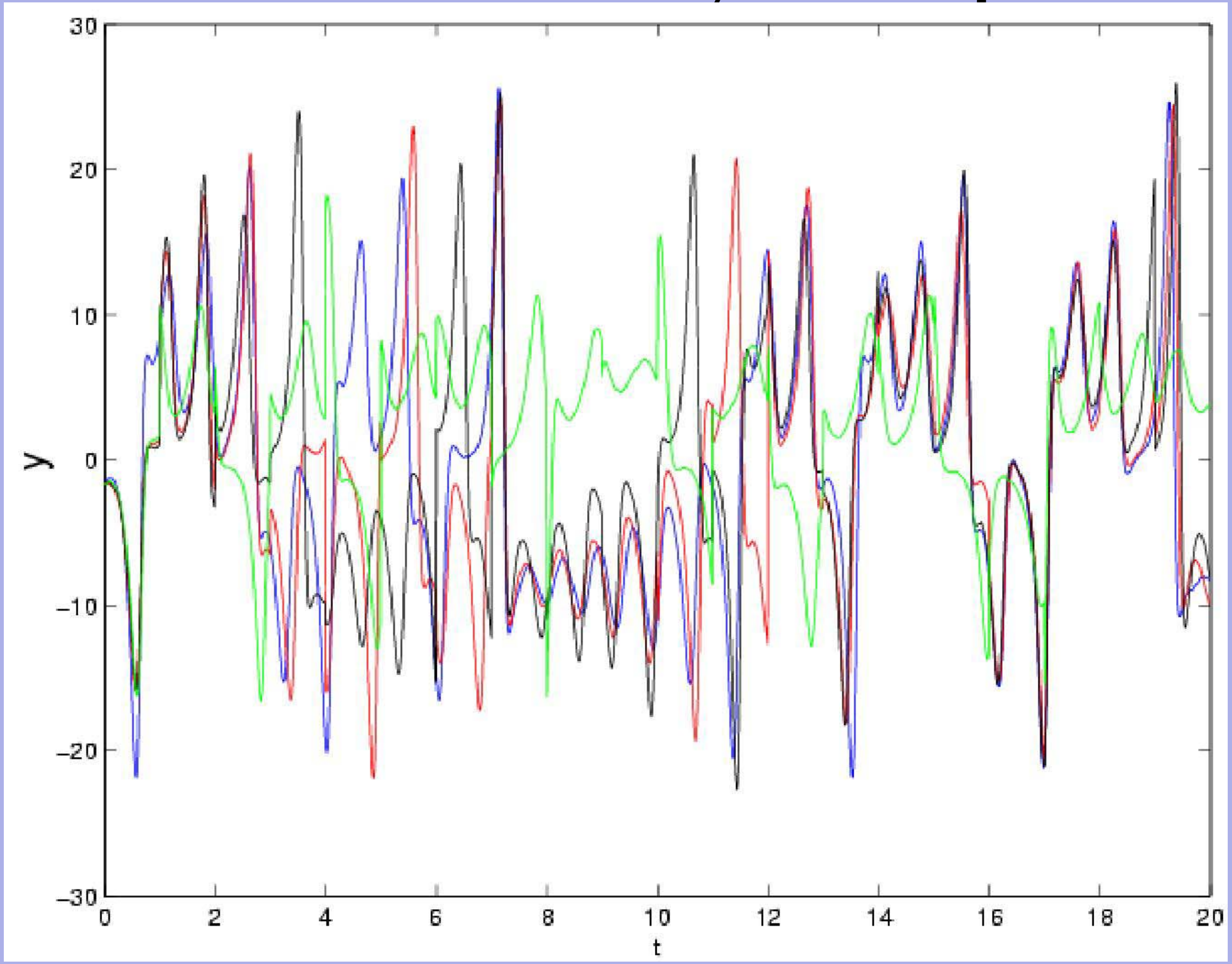
Ensemble Kalman Filter (1000)

SIR Filter (250)

SIR Filter (1000)

* observations

Lorenz model y component (not obs)



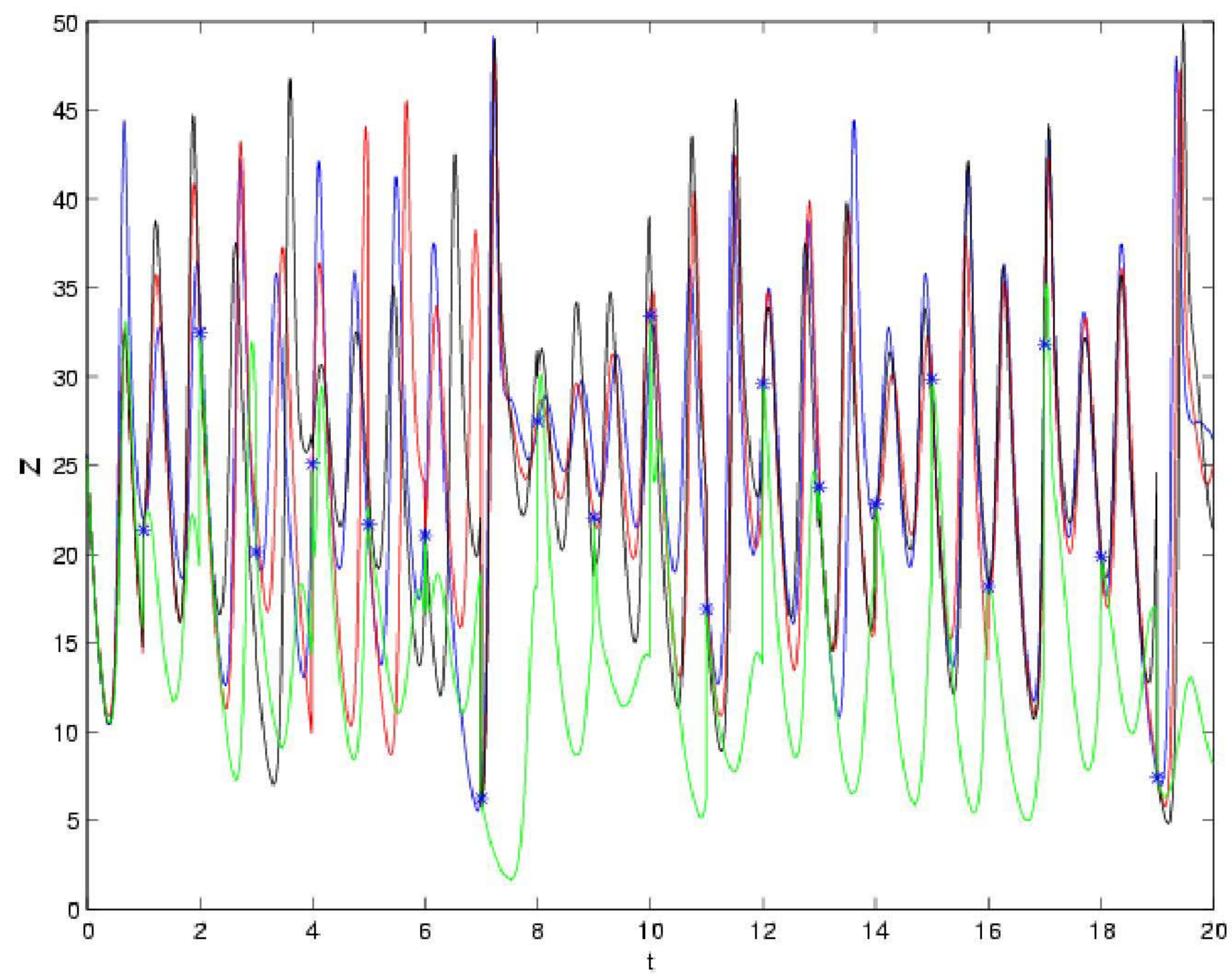
Ensemble Kalman Filter (1000)

SIR Filter (250)

SIR Filter (1000)

* observations

Lorenz model z component (obs)



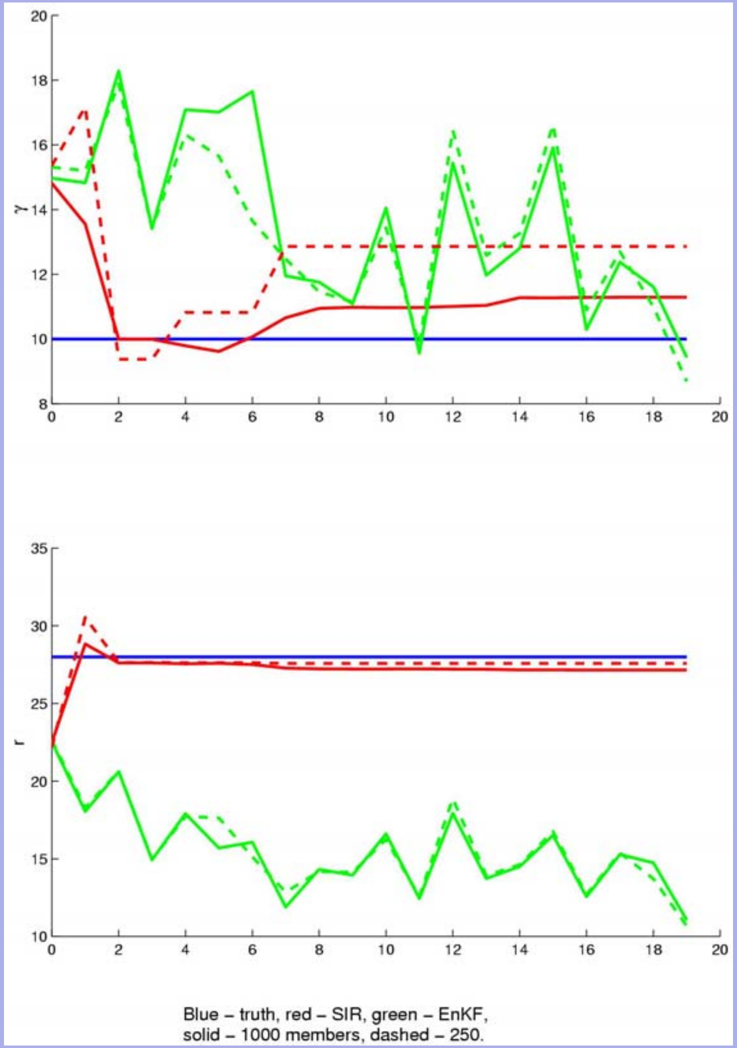
Ensemble Kalman Filter (1000)

SIR Filter (1000)

SIR Filter (250)

* observations

Lorenz model parameter values



Ensemble Kalman Filter (1000)

SIR Filter ----- (1000)

SIR Filter - - - (250)

true values

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